

Family Unification via Quasi-Nambu-Goldstone Fermions in String Theory

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Abstract

Some of the supersymmetric nonlinear sigma models on exceptional groups (E_7 or E_8) are known to yield almost minimal necessary content of matter fields for phenomenological model building, including three chiral families and a Higgs multiplet. We explore a possible realization of such a family unification in heterotic string theory, where the spontaneously broken symmetry that we focus on is the one associated with the change of global orientation of the spin bundle embedded in the gauge bundle, which cannot be ignored in the presence of a magnetic source of the B field. We show in a simple model that it indeed gives rise to chiral matter fields on the defect similarly to the domain-wall fermions, and compare the chiral zero mode spectrum with the sigma model. The setup is reduced to a system similar to that postulated in the orbifold GUT or the grand gauge-Higgs unification model, where the origin of \mathbf{Z}_2 identification is accounted for as arising from the bolt of the Atiyah-Hitchin manifold.

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I. INTRODUCTION

One of the long-standing questions in particle physics is why there are three families of quarks and leptons in Nature. While grand unification successfully unifies gauge interactions and gathers quarks and leptons in one generation into an irreducible representation, the repetitive structure of fermions with identical quantum numbers has been a mystery for decades.

For almost thirty years superstring theory has been long thought of as a promising framework for describing the ultra-violet dynamics of all interactions in a unified way, yet it is still not clear precisely how the standard model is realized in it. One of the major obstacles in constructing realistic phenomenological models in string theory is the appearance of numerous moduli, which are ever present in any smooth supersymmetric compactifications. Roughly speaking, moduli are parameters of the internal six-dimensional manifold and any structures thereof, and give rise to massless scalars in the low-energy field theory. They are problematic for phenomenological applications of string theory for the following reasons:

First, they may cause the cosmological moduli problem [1]. If they are present to date, their masses must be light enough in order for their energy density not to exceed the observed cosmological bound. On the other hand, if they are assumed to have decayed, there is a risk that they may spoil the success of the BBN scenario or the released entropy may dilute the baryon asymmetry produced by that time, depending on the time they decay. These problems might be avoided by modifying the conventional cosmological scenario.

The second reason concerns the moduli stabilization. There are many ways to realize standard-like models in string theory; each option has advantages and drawbacks. Moduli stabilization is the most successful in type IIB string theory, where the KKLT [2] and LARGE volume scenarios [3] are well known. However, it was pointed out that [4] to combine the moduli stabilization sector and the standard model sector is sometimes not straightforward. The D-brane models [5] also have a difficulty that the up-type Yukawa couplings can be generated only nonperturbatively [6, 7]. On the other hand, the model building based on $E_8 \times E_8$ heterotic string is the oldest and still attractive approach, but to completely fix all the moduli is harder than type II theories [8–12].

Finally, last but not least, the numerous moduli lead to many different possible string vacua - the landscape - which is also due to the existence of numerous possible compact-

ification manifolds. This is a problem since it prevents us from uniquely determining the high-energy extension of the standard model. None of the string theory realizations of a standard-like model known to date do not *explain* why we observe three families. Although the anthropic principle is indeed a constraint from the undeniable “observation” that we live in the Universe, it would not be strong enough to single out, for instance, the correct type of Calabi-Yau (if any) or the precise values of fluxes from a huge number of possibilities. The statistical approach [13] has shown no evidence of probabilistic dominance of standard-like models. On the contrary, standard-like models are rather rare [14].

In field theory (as opposed to string theory), an interesting proposal had been put forward in [15] long time ago, even before superstring theories have come to be known, where it was suggested that the quarks and leptons could be understood as quasi-Nambu-Goldstone fermions [16] of a supersymmetric nonlinear sigma model [15, 17–20, 22]. The idea that the family structure comes from a group theory is an old one [23] [62]. The advantage of the coset sigma model approach is that the associated quasi-Nambu-Goldstone fermions are typically chiral. Remarkably enough, it was shown by Kugo and Yanagida [18] that the sigma model based on $E_7/(SU(5) \times SU(3) \times U(1))$ yields precisely *three* sets of chiral superfields transforming in $\mathbf{10} \oplus \bar{\mathbf{5}}$ in addition to a single $\mathbf{5}$ of $SU(5)$ as the target space, the former of which may be identified as three generations of quark and lepton supermultiplets, and the latter a Higgs multiplet, respectively. Thus, the Kugo-Yanagida model offers a chance to realize almost minimal necessary matter content for an $SU(5)$ GUT in an amazingly economical way[63].

In this paper we propose a scenario for realizing such a family unification in *heterotic string theory* [24]. The spontaneously broken symmetry that we focus on is the one associated with the change of global orientation of the spin bundle embedded in the gauge bundle, which becomes relevant [25, 26] in the presence of a magnetic source of the B field, that is, the NS5-branes [27, 28]. It has been known for some time that the symmetric 5-brane in heterotic string theory supports charged chiral fermions [26, 28], which can be regarded as the quasi-NG fermions associated with the spontaneously chosen gauge field configuration of the brane. We consider a system of two stacks of intersecting 5-branes in the $E_8 \times E_8$ theory, which is expected to realize the Kähler coset $E_8/(E_6 \times H)$ for some subgroup H containing a $U(1)$ factor. We will perform an explicit computation of the Dirac zero modes on a compactified smeared intersecting 5-brane background to find two chiral and one anti-

chiral zero modes in the **27** representation of E_6 . This agrees with the chiral spectrum of the supersymmetric $E_8/(E_6 \times H)$ sigma model, confirming the expectation.

For the purpose of obtaining more realistic cosets, we also perform a similar computation in the background with a constant $U(1)$ Wilson line in addition to the $SU(3)$ gauge configuration. This time, unfortunately, a pair of chiral and anti-chiral modes turn out to appear and are asymmetrically localized on different branes, being unsatisfactory as a realization of the $E_8/(SO(10) \times H)$ coset. We will speculate about the non-standardly embedded NS5-branes that may lead to realistic Kähler cosets having three generations (with a possible anti-chiral generation and or a pair of chiral and anti-chiral generations).

The idea that chiral fermions are localized on a defect is a familiar one (e.g. [29, 30]), so it is not a crazy thought. On the other hand, this setup is quite different from the conventional smooth Calabi-Yau compactifications. It is somewhat similar to the old setup by Witten [31], but the crucial difference is that our setup contains NS5-branes. Of course, NS5-branes are difficult objects to deal with; the supergravity solution for NS5-branes develops an infinite throat, and the dilaton blows up as it goes down the throat. The microscopic description is less understood than D-branes. However, the idea is that one might capture their *low-energy* dynamics by a nonlinear sigma model, just in the same way as one can describe pions even if one did not know anything about QCD.

The organization of this paper is as follows. In section II, we give a brief review of the supersymmetric nonlinear sigma models with a target space being a Kähler coset of some exceptional Lie group. In section III, we turn to the construction of an NS5-brane system that may realize the symmetry breaking. We first show what symmetry in heterotic string theory is spontaneously broken in the presence of NS5-branes. We then solve explicitly the gaugino Dirac equation on a compactified smeared background and compare the chiral zero mode spectrum with that of the corresponding sigma model. The setup is reduced to a system similar to that postulated in the orbifold GUT or the (grand) gauge-Higgs unification. We also suggest a possible interpretation of the negative tension branes in heterotic string theory as a superposition of T-duals of the Atiyah-Hitchin manifold. In section IV, a similar computation is performed with a constant Wilson line included in one of the transverse directions. We conclude in section V with a summary and discussion on possible future directions. Some useful facts about E_8 and conventions for the gamma matrices are collected in two appendices.

II. EXCEPTIONAL COSET SUPERSYMMETRIC NONLINEAR SIGMA MODELS

A nonlinear sigma model on a group coset G/H describes the low-energy dynamics of Nambu-Goldstone (NG) bosons which arise when a global symmetry G is spontaneously broken to its subgroup H . In supersymmetric nonlinear sigma models those NG bosons are accompanied by their super-partners called quasi-Nambu-Goldstone (qNG) fermions. In four dimensions the target space of an $\mathcal{N} = 1$ nonlinear sigma model is a Kähler manifold, and the chiralities of the qNG fermions can readily be determined by examining their “Y-charges” [22] of the Kähler coset, as we briefly review below.

Let G be a compact, semi-simple group, and H be its subgroup. The classic theorem of Borel asserts that the coset space G/H is Kähler if and only if H is a centralizer of some torus subgroup of G , that is, H is the group consisting of all elements that commute with some $U(1)^n$ subgroup of G . This means that if G/H is Kähler, any element in G/H has nonzero charge for some $U(1)$ subgroup. Therefore, one can take some linear combination of the $U(1)$ charges so that the complexified Lie algebra of G/H is decomposed into a direct sum of positive- and negative-charge eigenspaces. We fix some such a combination and call it “Y-charge”, following [22]. Let X^I ($I = 1, \dots, k$) be generators having negative Y-charge, with k being half the real dimensions of G/H , and consider the “BKMU variable”

$$\xi(\phi) \equiv e^{\phi^I X^I}, \quad (1)$$

where ϕ^I 's are the set of chiral superfields parameterizing the Kähler coset. Then it was shown that the Kähler potential can be expressed in terms of $\xi(\phi)$ [19, 22] (see also [32, 33] for earlier discussions), and a certain H -invariant projection operator η acting on the representation vector space. Moreover, the correspondence between the Y-charge and the complex structure of the Kähler manifold is one-to-one. Therefore, in order to determine the chiral spectrum of a given coset, we have only to choose a suitable combination of $U(1)$ charges as the Y-charge and examine which generators have negative Y-charges.

We now turn to the actual decompositions in the relevant examples:

A. $E_7/(SU(5) \times SU(3) \times U(1))$

This is the original coset space considered by Kugo and Yanagida [18]. The adjoint **133** of E_7 is decomposed into a sum of irreducible representations of $SU(5) \times SU(3)$ as follows:

$$\begin{aligned} \mathbf{133} = & (\mathbf{24}, \mathbf{1})_0 \oplus (\mathbf{1}, \mathbf{8})_0 \oplus (\mathbf{1}, \mathbf{1})_0 \oplus (\mathbf{5}, \bar{\mathbf{3}})_4 \oplus (\bar{\mathbf{5}}, \mathbf{3})_{-4} \\ & \oplus (\mathbf{5}, \mathbf{1})_{-6} \oplus (\bar{\mathbf{5}}, \mathbf{1})_6 \oplus (\mathbf{10}, \bar{\mathbf{3}})_{-2} \oplus (\bar{\mathbf{10}}, \mathbf{3})_2, \end{aligned} \quad (2)$$

where the subscript numbers indicate the $U(1)$ charges of the representations they follow. The charge normalization is taken so that it coincides with $-h_\sharp$ (A10). (The minus sign is needed to agree with the conventions of the chirality in the literature.) The representations in the first line form an $SU(8)$ subalgebra, whereas those in the second line do a rank-4 anti-symmetric tensor representation **70** of $SU(8)$, the familiar realization of E_7 in supergravity [34]. Those which have negative charges are

$$(\bar{\mathbf{5}}, \mathbf{3})_{-4}, (\mathbf{10}, \bar{\mathbf{3}})_{-2}, (\mathbf{5}, \mathbf{1})_{-6}. \quad (3)$$

Therefore, the qNG fermions of this model are three sets of $\mathbf{10} \oplus \bar{\mathbf{5}}$ and a single **5** of $SU(5)$.

B. $E_8/(SU(5) \times SU(3) \times U(1)^2)$

It has also been known for some time that the net number of chiral qNG fermions does not change if the coset group is extended from E_7 to E_8 with an appropriate choice of the $U(1)$ Y charge. The relevant decompositions are

$$\begin{aligned} E_8 & \supset E_7 \times SU(2) \\ \mathbf{248} & = (\mathbf{133}, \mathbf{1}) \oplus (\mathbf{56}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{3}) \end{aligned} \quad (4)$$

$$\begin{aligned} E_7 & \supset SU(5) \times SU(3) \times U(1)_{-h_\sharp} \\ \mathbf{56} & = (\mathbf{5}, \mathbf{3})_{-1} \oplus (\bar{\mathbf{5}}, \bar{\mathbf{3}})_1 \oplus (\mathbf{1}, \mathbf{3})_5 \oplus (\mathbf{1}, \bar{\mathbf{3}})_{-5} \oplus (\mathbf{10}, \mathbf{1})_3 \oplus (\bar{\mathbf{10}}, \mathbf{1})_{-3}, \end{aligned} \quad (5)$$

where the $U(1)$ factor contained in E_7 is specifically called $U(1)_{-h_\sharp}$ so as to distinguish from the one coming from $SU(2)$. The decomposition of $(\mathbf{133}, \mathbf{1})$ is the same as (2). The E_7 singlet $(\mathbf{1}, \mathbf{3})$ has h_\sharp charge 0.

If $U(1)_{h_\sharp}$ is again chosen to be the Y charge, then in addition to (3) the new qNG fermions $(\mathbf{5}, \mathbf{3})_{-1}$, $(\mathbf{1}, \bar{\mathbf{3}})_{-5}$ and $(\bar{\mathbf{10}}, \mathbf{1})_{-3}$ (which are also doublets of $SU(2)$) arise, destroying

the excellent similarity to the pattern of the observed chiral fermions. However, let h_8 be the generator of the $U(1)$ subalgebra of the $SU(2)$ such that a pair in the doublet $\mathbf{2}$ have eigenvalues ± 1 of opposite signs, defining

$$Y = -h_{\#} + c h_8 \quad (6)$$

with a real constant c . Then, for example, if c is taken to be 4, then (5) is replaced by

$$E_8 \supset SU(5) \times SU(3) \times U(1)_Y$$

$$(\mathbf{56}, \mathbf{2}) = (\mathbf{5}, \mathbf{3})_{-1\pm 4} \oplus (\bar{\mathbf{5}}, \bar{\mathbf{3}})_{1\pm 4} \oplus (\mathbf{1}, \mathbf{3})_{5\pm 4} \oplus (\mathbf{1}, \bar{\mathbf{3}})_{-5\pm 4} \oplus (\mathbf{10}, \mathbf{1})_{3\pm 4} \oplus (\bar{\mathbf{10}}, \mathbf{1})_{-3\pm 4}, \quad (7)$$

where the subscripts \pm are understood to mean the direct sum of these spaces. In this way the values of Y split in each doublet. The negative Y -charge components are

$$(\mathbf{5}, \mathbf{3})_{-5}, \quad (\bar{\mathbf{5}}, \bar{\mathbf{3}})_{-3}, \quad (\mathbf{1}, \bar{\mathbf{3}})_{-5\pm 4}, \quad (\mathbf{10}, \mathbf{1})_{-1}, \quad (\bar{\mathbf{10}}, \mathbf{1})_{-7}, \quad (8)$$

hence nonchiral after the $SU(3)$ breaking. This leaves the same chiral spectrum as that of the previous example $E_7/(SU(5) \times SU(3) \times U(1))$.

C. $E_8/(SO(10) \times SU(3) \times U(1))$

In this case the decomposition reads

$$\begin{aligned} \mathbf{248} = & (\mathbf{45}, \mathbf{1})_0 \oplus (\mathbf{16}, \mathbf{1})_3 \oplus (\bar{\mathbf{16}}, \mathbf{1})_{-3} \oplus (\mathbf{1}, \mathbf{1})_0 \\ & \oplus (\mathbf{16}, \mathbf{3})_{-1} \oplus (\mathbf{10}, \mathbf{3})_2 \oplus (\mathbf{1}, \mathbf{3})_{-4} \\ & \oplus (\bar{\mathbf{16}}, \bar{\mathbf{3}})_1 \oplus (\mathbf{10}, \bar{\mathbf{3}})_{-2} \oplus (\mathbf{1}, \bar{\mathbf{3}})_4 \\ & \oplus (\mathbf{1}, \bar{\mathbf{8}})_0, \end{aligned} \quad (9)$$

where the $U(1)$ charges shown as subscripts are those of $3h_{\perp}$ (A15). Each line is E_6 irreducible. One can read off from this decomposition that the negative charge components are

$$(\bar{\mathbf{16}}, \mathbf{1})_{-3}, (\mathbf{16}, \mathbf{3})_{-1}, (\mathbf{1}, \mathbf{3})_{-4}, (\mathbf{10}, \bar{\mathbf{3}})_{-2}, \quad (10)$$

which contain three chiral $\mathbf{16}$ generations and one $\bar{\mathbf{16}}$ anti-chiral generation. It has been proved that, even if the $SU(3)$ is further broken to $U(1)^2$, one can not obtain four chiral (instead of three chiral plus one anti-chiral) fermions for any choice of the $U(1)$ Y charge [22].

D. $E_8/(E_6 \times SU(2) \times U(1))$

The final example is the coset, which, as we will show in later sections, is to be realized by a system of two stacks of intersecting heterotic 5-branes with the standard embedding. The decomposition reads

$$248 = (\mathbf{78}, \mathbf{1})_0 \oplus (\mathbf{27}, \mathbf{2})_{-1} \oplus (\mathbf{27}, \mathbf{1})_2 \oplus (\overline{\mathbf{27}}, \mathbf{2})_1 \oplus (\overline{\mathbf{27}}, \mathbf{1})_{-2} \oplus (\mathbf{1}, \mathbf{1})_0. \quad (11)$$

III. SPONTANEOUS SYMMETRY BREAKING BY NS5-BRANES IN HETEROtic STRING THEORY

A. What symmetry is broken spontaneously?

In the previous section we have seen that some of the supersymmetric nonlinear sigma models on E_8 coset spaces have attractive chiral matter spectra for particle physics model building. In this section we explore the possibility of realizing such nonlinear sigma models by using $NS5$ -branes in $E_8 \times E_8$ heterotic string theory.

First of all, if these sigma models are realized in any setup, some symmetry must be broken spontaneously. What symmetry is broken spontaneously in heterotic string theory? To understand this point, let us recall the well-known 5-brane solution [27, 28] in heterotic string theory:

$$\begin{aligned} g_{ij} &= \eta_{ij} \quad (i, j = 0, 1, \dots, 5), \\ g_{\mu\nu} &= e^{2\phi} \delta_{\mu\nu} \quad (\mu, \nu = 6, \dots, 9), \\ e^{2\phi} &= e^{2\phi_0} + \frac{n\alpha'}{x^2}, \\ H_{\mu\nu\lambda} &= -\epsilon_{\mu\nu\lambda}{}^\rho \partial_\rho \phi, \\ A_\mu^{\alpha\beta} &= 2\rho^2 \sigma^{\alpha\beta}{}_{\mu\lambda} \cdot \frac{x^\lambda}{x^2(x^2 + \rho^2)}, \end{aligned} \quad (12)$$

where $x^2 \equiv \sum_{\mu=6}^9 (x^\mu)^2$, and $\epsilon^{\mu\nu\lambda\rho}$ is the (undensitized) completely antisymmetric tensor. All other components of H vanish.

They satisfy the equations of motion of the low-energy effective supergravity of heterotic string theory to leading order in the α' expansion. The bosonic part of the Lagrangian is

given by

$$\mathcal{L} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left\{ R(\omega) - \frac{1}{3} H_{MNP} H^{MNP} + 4(\partial_M \phi)^2 - \alpha' \left(\frac{1}{30} \text{Tr}(F_{MN} F^{MN}) - R_{MNAB}(\omega_+) R^{MNB A}(\omega_+) \right) \right\} \quad (13)$$

to this order. If all A_M is set to zero in (12), the set of field configurations is reduce to the neutral $NS5$ -brane solution for type II theories. The gauge field configuration (12) is higher-order in α' , and can be obtained by the so-called standard embedding

$$A_M^{AB} = \omega_M^{AB} + H_M^{AB} \quad (14)$$

with ω_M^{AB} being the spin connection. It is a solution of the well-known heterotic Bianchi identity [35]

$$dH = \alpha' \left(\text{tr} R(\omega_+) \wedge R(\omega_+) - \frac{1}{30} \text{Tr} F \wedge F \right), \quad (15)$$

whose right hand side is required by the celebrated Green-Schwarz mechanism of anomaly cancellation.

Now the point is that the way of embedding the (generalized) spin connection ($= \omega + H$) in E_8 is not unique. In the configuration (12), a particular $SU(2)$ subalgebra of E_8 is chosen to be set equal to the spin connection, thereby the rotational symmetry of the gauge orientation is spontaneously broken. The number of moduli is that of generators which nontrivially act on the gauge configuration, hence

$$248 - 133 = 115, \quad (16)$$

where 133 is the dimension of the commutant, E_7 , of $SU(2)$ in E_8 . It was pointed out [28] that these 115 moduli, together with 4 translation and one scale moduli, in all 120 give rise to massless scalars on the 5-brane to form 30 six-dimensional hypermultiplets in heterotic string theory.

The gauge configuration of the symmetric 5-brane (12) has

$$\frac{1}{480\pi^2} \text{Tr} F \wedge F = 1, \quad (17)$$

and hence may be regarded as a gauge instanton in the transverse four-dimensional space. In a flat space, the dimensions of moduli parameters of gauge instantons are given by

$$4C(G)k - d(G), \quad (18)$$

where $C(G)$ and $d(G)$ are respectively the quadratic Casimir and dimension of the compact gauge group G , if the instanton number k is large enough [36]. If $G = E_8$, it is valid if $k \geq 3$. Although this formula was derived by using the Atiyah-Singer index theorem, the number was also given an interpretation [36] as that required to label (in addition to the scales, positions and sizes) the group orientations of k $SU(2)$ instantons embedded in the group G . In the E_8 case, the Casimir is 30, and the number is $120k - 248$, the number 120 being in agreement with the above 5-brane moduli.

However, there is a crucial difference between the moduli of the heterotic 5-brane and those of gauge instantons in a flat space. If the instanton number k is one (as in (17)), the moduli formula (18) is not correct, but a single E_8 instanton has only 5 moduli in the four-dimensional Euclidean space [36]. This means that the apparent 115 variations are pure gauge modes and do not change physics. On the other hand, unlike the flat-space gauge instantons, these gauge variations in heterotic string theory *do* change physics by leaving their traces on the locations of the magnetic source of the B field, that is, the 5-branes. This is due to the Green-Schwarz counter term contained in the heterotic Lagrangian [25, 26].

According to conventional wisdom in field theory, one might think that there would be no such moduli because the NG modes associated with a break-down of a *local* symmetry are eaten by the gauge fields. This indeed also happens here; let $\delta_{\Lambda^{E_7}}$, $\delta_{\Lambda^{(56,2)}}$ and $\delta_{\Lambda^{SU(2)}}$ be E_8 gauge transformations corresponding to the decomposition (4), with gauge parameter functions $\Lambda^{E_7}(x^i)$, $\Lambda^{(56,2)}(x^i)$ and $\Lambda^{SU(2)}(x^i)$ depending only on the coordinates parallel to the brane, then $\delta_{\Lambda^{(56,2)}}A_\mu$ and $\delta_{\Lambda^{SU(2)}}A_\mu$ are nonzero and satisfy the linearized equations of motion, which can be undone by counter gauge transformations on A_i . This is the ordinary Higgs mechanism, where the gauge components in the transverse dimensions play the role of adjoint Higgs fields, consequently the gauge symmetry is broken to E_7 . The gauge parameter functions can also depend on the transverse coordinates x^μ . In this case, the analogous gauge deformations are eaten by the higher Kaluza-Klein gauge fields.

However, this is not the end of the story; the gauge variation of the Green-Schwarz counter term (with exact terms neglected) is given by [37]

$$\begin{aligned}\delta(-dB \wedge X_7) &= -\delta(dB) \wedge X_7 - dB \wedge \delta(X_7) \\ &= -d(\omega_{2Y}^1 - \omega_{2L}^1) \wedge X_7 - dB \wedge dX_6^1.\end{aligned}\tag{19}$$

The first term is a well-known contribution to complete the Green-Schwarz mechanism for

anomaly cancellation in heterotic string theory, while the second term usually vanishes by partial integration. However, if there is a magnetic source of the B field, the latter also gives a nonzero result since

$$d^2 B \propto \delta^4(x). \quad (20)$$

Therefore, unlike the unitary gauge in the usual Higgs mechanism, gauge variations of the heterotic 5-brane configuration are not completely absorbed into the massive gauge fields.

The clearest evidence for the existence of chiral fermions on the brane is provided by the anomaly cancellation argument [26, 38]. If there are 120 NG bosons on the brane, their quasi NG fermions produce gauge and gravitational anomalies. It has been shown in [26] that the sum of these anomalies and the anomaly inflow (cf. (19),(20)) turns out to be in a factorized form just like the ordinary ten-dimensional bulk anomalies, and can be similarly cancelled by a Green-Schwarz counter term on the brane. If it were not for the 115 zero modes from the spontaneously broken gauge rotations, the sum of anomalies would fail to factorize, and the successful cancellation argument would be invalidated.

To recap, an NS5-brane in heterotic string theory has a gauge instanton in the transverse space and spontaneously chooses a particular embedding of the spin connection into the gauge connection. The associated NG modes are localized on the brane together with their superpartners, whose anomalies consistently match with the anomaly inflow from the bulk. This opens up a possibility of realizing the nonlinear sigma models on the E_8 cosets discussed in section II.

B. Intersecting 5-branes and $E_8/(E_6 \times SU(2) \times U(1))$ sigma model

In the previous section, we have seen that a stack of parallel NS5-branes spontaneously breaks the translation, scale and gauge rotation symmetries to yield localized modes on the magnetic-source locus of the B field. In order to have, on the other hand, a four-dimensional theory with $N = 1$ supersymmetry, we consider [25, 26, 39] two stacks of intersecting NS5-branes. In four dimensions there is no pure gravitational anomaly, nor is there gauge anomaly for E_6 . However, each of 5-branes should support chiral fermions so that their anomalies cancel against the anomaly inflow from the bulk. The two 5-branes have four common space-time directions, one is extended in $(x^0, x^5, x^6, x^7, x^8, x^9)$ directions and

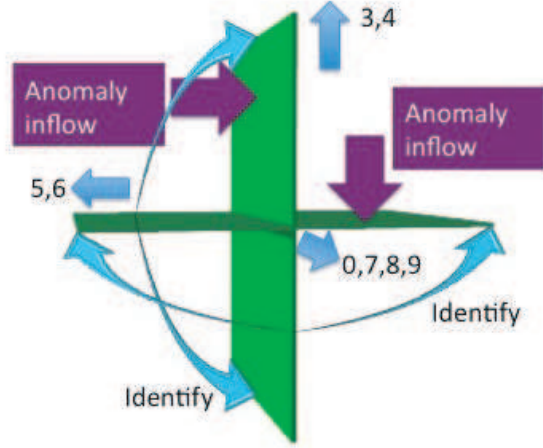


FIG. 1: Two stacks of intersecting NS5-branes. The intersection is four-dimensional. Relatively transverse directions are compactified on a torus by periodic identifications. The two overall transverse directions (x^1 and x^2) are not depicted here.

the other in $(x^0, x^3, x^4, x^7, x^8, x^9)$ directions. Thus we compactify the relatively transverse directions (x^3, x^4, x^5, x^6) on a 4-torus. The configuration that we consider is also smeared in one of the overall transverse directions x^2 , which is further compactified on S^1 . In this way we end up with a system of four-dimensional space-time (x^0, x^7, x^8, x^9) with a single extra dimension x^1 (FIG.1; see also FIG4).

Supergravity solutions for two intersecting NS5-branes localized in all but one of the transverse directions are known [40]. They were obtained in type II theories and hence are gauge neutral, but may also be promoted to leading-order heterotic supergravity solutions by the standard embedding. However, later we are going to analyze the gaugino Dirac equation on the intersecting NS5-brane background, and this solution is too complicated for our purposes. Instead, we consider the well-known smeared solution in all except one transverse direction [41], which is very simple to deal with. One of its virtue is that, unlike the original parallel 5-brane solution (12) which develops a throat geometry where the theory is strongly coupled, the domain-wall type smeared solution has a finite string coupling even near the brane locus. This solution is also interesting since, as we mentioned above, it can be reduced to a Randall-Sundrum-like [42][64] extra dimension model with a GUT gauge group, a setup similar to that postulated in the orbifold GUT or the grand gauge-Higgs

unification scenario [43, 44].

In the string frame, the (gauge) neutral solution is given by

$$\begin{aligned} ds^2 &= \eta_{ij} dx^i dx^j + h(x)^2 \delta_{\mu\nu} dx^\mu dx^\nu + h(x) \delta_{\mu'\nu'} dx^{\mu'} dx^{\nu'} \\ e^{2\phi} &= h(x)^2, \\ H_{234} &= H_{256} = \frac{h'(x)}{2} \end{aligned} \quad (21)$$

with

$$h(x) = g_0 \left(1 - \frac{|x|}{L} \right), \quad (22)$$

where i, j run over $\{0, 7, 8, 9\}$, μ', ν' do over $\{3, 4, 5, 6\}$ and μ, ν take values $\{1, 2\}$. All the fields depend only on the x^1 coordinate, and we write $x \equiv x^1$.

The indices of H are the curved ones. Other components of H are zero. g_0 and L are real, positive constants; g_0 is the value of the string coupling at $x = 0$, whereas L is the scale of the transverse dimension.

These set of configurations describe two intersecting NS5-branes, stretched along the x^0 -, x^5 -, x^6 -, x^7 -, x^8 - and x^9 -axes and the x^0 -, x^3 -, x^4 -, x^7 -, x^8 - and x^9 -axes, respectively. They satisfy the equations of motion and preserve 1/4 of supersymmetry if

$$|x| < L, \quad (23)$$

so that the function $h(x)$ is positive; the solution is well-defined only in a finite interval with a finite proper distance. In order to avoid the occurrence of a negative $h(x)$ region, we consider [25, 26, 39] a periodic array of (21) by taking

$$h(x) = g_0 \left(1 - \frac{|x - 2nL|}{L} \right) \quad \text{if} \quad (2n-1)L < x \leq (2n+1)L \quad \text{for some } n \in \mathbf{Z} \quad (24)$$

(FIG.2), and periodically identify the $x(=x^1)$ space by a relation $x \sim x + 2L$. Thus the x^1 direction is compactified on a circle. (Later we argue that this circle should be \mathbf{Z}_2 orbifolded further.)

The function $h(x)$ has two singularities, at which magnetically charged objects with opposite H charges are located. Note that while the one at $x = 0$ is an ordinary intersecting 5-branes with positive tension, the other at $x = \pm L$ has negative tension. Later, in section III E, we will discuss how such an orientifold-like object can be understood in heterotic string theory.

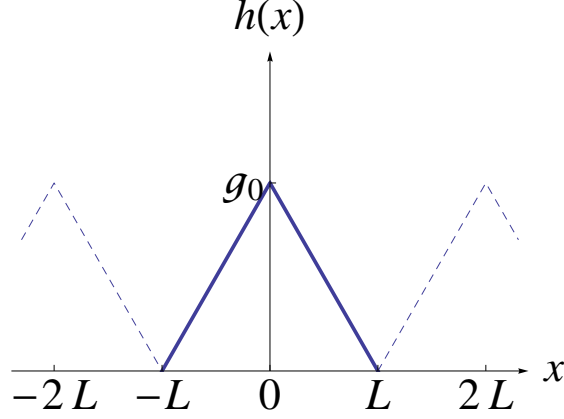


FIG. 2: The periodic function $h(x)$.

The solution (21) preserve 1/4 of supersymmetry. One of the Killing spinor equations is the gravitino variation equation:

$$\delta\psi_M = \left(\partial_M + \frac{1}{4}(\omega - H)_M{}^{AB}\Gamma_{AB} \right) \epsilon = 0. \quad (25)$$

The existence of a non-trivial Killing spinor implies that the $SO(6)$ connection $\omega - H$ is actually in $SU(3)$, as is easily verified. Also it can be checked that the other combination $\omega + H$ is also in a different $SU(3)$ sub-algebra of $SO(6)$. Explicitly,

$$\begin{aligned} \omega_1 + H_1 &= 0, \\ \omega_2 + H_2 &= -i \frac{h'}{2h^2} \cdot \frac{3\lambda_3 + \sqrt{3}\lambda_8}{2} \otimes \sigma_2, \\ \omega_3 + H_3 &= -i \frac{h'}{2h^2} \cdot \lambda_2 \otimes \mathbf{1}, \\ \omega_4 + H_4 &= -i \frac{h'}{2h^2} \cdot \lambda_1 \otimes \sigma_2, \\ \omega_5 + H_5 &= -i \frac{h'}{2h^2} \cdot \lambda_5 \otimes \mathbf{1}, \\ \omega_6 + H_6 &= -i \frac{h'}{2h^2} \cdot \lambda_4 \otimes \sigma_2, \end{aligned} \quad (26)$$

where the three of the latter two indices of ω and H are regarded as the matrix indices. $\lambda_1, \dots, \lambda_8$ are the Gell-Mann matrices, and $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices.

Each 2×2 block of the 6×6 matrices (26) is either $\mathbf{1}$ or $i\sigma_2$. In mapping $SO(6)$ to $SU(3)$, $\mathbf{1}$ can be simply dropped, while there is a sign ambiguity in reducing $i\sigma_2$ to $\pm i$; flipping this sign corresponds to going from the $\mathbf{3}$ representation to the $\bar{\mathbf{3}}$ representation, and vice versa. Denoting this sign by $s = \pm$, the $SU(3)$ configuration obtained by the standard embedding

is given by

$$\begin{aligned}
A_1 &= 0, \\
A_2 &= -i \frac{h'}{2h^2} \cdot s \frac{3\lambda_3 + \sqrt{3}\lambda_8}{2}, \\
A_3 &= -i \frac{h'}{2h^2} \cdot \lambda_2, \\
A_4 &= -i \frac{h'}{2h^2} \cdot s\lambda_1, \\
A_5 &= -i \frac{h'}{2h^2} \cdot \lambda_5, \\
A_6 &= -i \frac{h'}{2h^2} \cdot s\lambda_4.
\end{aligned} \tag{27}$$

We now consider the moduli. The gauge configuration (27) chooses a particular $SU(3)$ sub-algebra of E_8 . The gauge rotation moduli consist of the gauge rotations that change this configuration. They are those in

$$(\mathbf{1}, \mathbf{8}), \quad (\mathbf{27}, \mathbf{3}), \quad (\overline{\mathbf{27}}, \mathbf{\bar{3}}) \tag{28}$$

as representations of $E_6 \times SU(3)$. They are *real* coordinates of the coset space E_8/E_6 , which is not Kähler. However, (28) can be understood as arising from a two-step breaking, in which E_8 is first broken to $E_6 \times U(1) \times U(1)$ (or $E_6 \times SU(2) \times U(1)$), giving a Kähler coset, and then the $U(1) \times U(1)$ (or $SU(2) \times U(1)$) symmetry is broken. The second breaking does not give rise to any E_6 -charged fields. Moreover, recently it has been found [45] that such extra $U(1)$ degrees of freedom are eliminated from the denominator if the sigma models are coupled to supergravity. Therefore, we consider the nonlinear sigma model on $E_8/(E_6 \times SU(2) \times U(1))$ (which is simpler than $E_8/(E_6 \times U(1) \times U(1))$; the chiral spectrum does not change whatever Y -charge is chosen) and examine what it implies about the chiralities of massless fermions on this background.

$E_8/(E_6 \times SU(2) \times U(1))$ is the last example discussed in section II, where we have seen that the sigma model has three chiral quasi NG fermions transforming nontrivially by the E_6 rotation. They are:

$$(\mathbf{27}, \mathbf{2}) \quad \text{and} \quad (\overline{\mathbf{27}}, \mathbf{1}). \tag{29}$$

Thus the sigma model predicts that there are two chiral and one anti-chiral generations transforming as $\mathbf{27}$ of E_6 . In the next section, we confirm this prediction by explicitly solving

the gaugino Dirac equation on the smeared intersecting 5-brane background. Essentially, this result was already obtained and announced in [25, 26]; in the present paper we will give a more complete discussion, including the precise boundary conditions set for the solutions and (non)normalizability [65].

C. Explicit computation of fermionic zero modes

The gaugino equation of motion of heterotic string theory can be compactly written in terms of a special combination of the spin and gauge connections (see e.g. [46]):

$$\Gamma^M D_M \left(\omega - \frac{H}{3}, A \right) \tilde{\chi} = 0, \quad (30)$$

where

$$\chi = e^\phi \tilde{\chi} \quad (31)$$

is the original gaugino in the Lagrangian.

$$\Gamma^i D_i \tilde{\chi} + \Gamma^\mu D_\mu \left(\omega - \frac{H}{3}, A \right) \tilde{\chi} = 0 \quad (32)$$

$$\tilde{\chi} = \tilde{\chi}_{4D}(x^i) \otimes \tilde{\chi}_{6D}(x^\mu) \quad (33)$$

See Appendix B for conventions for the gamma matrices used in this paper. In these conventions, a Weyl spinor with $\Gamma_{11} = +1$ is a linear combination of the spinors of the forms

$$\begin{bmatrix} * \\ * \\ 0 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} * \\ * \\ * \\ * \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 0 \\ * \\ * \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ * \\ * \\ * \\ * \end{bmatrix}. \quad (34)$$

The gaugino is Majorana-Weyl in ten dimensions. In the present notation a spinor ψ is said Majorana iff

$$B\psi^* = \psi, \quad (35)$$

where $*$ denotes the complex conjugate. Now suppose that we found χ_{6D} of the form

$$\begin{bmatrix} * \\ * \\ * \\ * \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

satisfying

$$\gamma^\mu D_\mu \left(\omega - \frac{H}{3}, A \right) \tilde{\chi}_{6D} = 0 \quad (36)$$

and χ_{4D} of the form $\begin{bmatrix} * \\ * \\ 0 \\ 0 \end{bmatrix}$ satisfying

$$\gamma_{4D}^i D_i \tilde{\chi}_{4D} = 0. \quad (37)$$

Then we can construct a Majorana spinor

$$\tilde{\chi} + B\tilde{\chi}^*, \quad \tilde{\chi} = \tilde{\chi}_{4D} \otimes \tilde{\chi}_{6D}. \quad (38)$$

which satisfies the equation (32). Thus, to look for gaugino zero modes, it is enough to solve the equation (36) for $\tilde{\chi}_{6D}$ of the form

$$\tilde{\chi}_{6D} = \begin{pmatrix} \tilde{\chi}_{6D}^+ \\ 0 \end{pmatrix}, \quad (39)$$

where $\tilde{\chi}^+$ is a four-component spinor. Note that if we consider $\tilde{\chi}_{6D}$ transforming as $(\mathbf{27}, \mathbf{3})$, each of the four components of $\tilde{\chi}_{6D}$ is tensored by a triplet vector of $SU(3)$ (and also a 27-plet of E_6).

We consider $\tilde{\chi}_{6D}$ in $(\mathbf{27}, \mathbf{3})$ that depends only on the $x^1 (\equiv x)$ coordinate. Writing

$$\gamma^1 = \begin{pmatrix} & -i\mathbf{1}_4 \\ i\mathbf{1}_4 & \end{pmatrix}, \quad \gamma^{\tilde{\alpha}} = \begin{pmatrix} & \tilde{\gamma}^{\tilde{\alpha}} \\ \tilde{\gamma}^{\tilde{\alpha}} & \end{pmatrix} \quad (\tilde{\alpha} = 2, \dots, 6), \quad (40)$$

the equation (36) becomes

$$\left(h^{-1} \begin{pmatrix} & -i\partial_x \\ i\partial_x & \end{pmatrix} + \frac{1}{4} \left(\omega - \frac{H}{3} \right)_{\tilde{\alpha}\beta\gamma} \gamma^{\tilde{\alpha}\beta\gamma} + \begin{pmatrix} & A_{\tilde{\alpha}} \tilde{\gamma}^{\tilde{\alpha}} \\ A_{\tilde{\alpha}} \tilde{\gamma}^{\tilde{\alpha}} & \end{pmatrix} \right) \tilde{\chi}_{6D}(x) = 0, \quad (41)$$

where the second term is also in the block off-diagonal form. This yields a simple first-order differential equation for $\tilde{\chi}_{6D}^+$:

$$\left(\frac{d}{dx} + \frac{h'(x)}{h(x)} M(s) \right) \tilde{\chi}_{6D}^+ = 0, \quad (42)$$

where the matrix $M(s)$ is a real, constant matrix given by

$$M(s) = \begin{pmatrix} \frac{3}{2} & 0 & -\frac{s}{2} & -\frac{1}{4} & -\frac{s}{2} & \frac{1}{2} & -s & \frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & \frac{3}{2} & 0 & -\frac{s}{2} & -\frac{1}{4} & 0 & -\frac{1}{2} & \frac{s}{2} & 0 & 0 & -\frac{1}{4} & 0 \\ -\frac{s}{2} & 0 & \frac{3}{2} & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 & \frac{s}{2} & 0 & 0 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{s}{2} & -\frac{1}{2} & \frac{3}{2} & 0 & \frac{s}{2} & -\frac{1}{4} & 0 & 0 & -s & \frac{1}{2} & 0 \\ -\frac{s}{2} & -\frac{1}{4} & 0 & 0 & \frac{3}{2} & 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{2} & \frac{s}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{4} & \frac{s}{2} & 0 & \frac{3}{2} & 0 & 0 & -\frac{1}{4} & 0 & 0 & \frac{s}{2} \\ -s & -\frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 & \frac{3}{2} & 0 & \frac{s}{2} & -\frac{1}{4} & \frac{s}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{s}{2} & 0 & 0 & -\frac{1}{4} & 0 & 0 & \frac{3}{2} & 0 & \frac{s}{2} & -\frac{1}{4} & 0 \\ 0 & 0 & \frac{s}{2} & 0 & 0 & -\frac{1}{4} & \frac{s}{2} & 0 & \frac{3}{2} & \frac{1}{2} & 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 & 0 & -s & -\frac{1}{2} & 0 & -\frac{1}{4} & \frac{s}{2} & \frac{1}{2} & \frac{3}{2} & 0 & -\frac{s}{2} \\ 0 & -\frac{1}{4} & 0 & \frac{1}{2} & \frac{s}{2} & 0 & \frac{s}{2} & -\frac{1}{4} & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & -\frac{1}{4} & 0 & 0 & \frac{s}{2} & -\frac{1}{2} & 0 & -\frac{1}{4} & -\frac{s}{2} & 0 & \frac{3}{2} \end{pmatrix}, \quad (43)$$

where $s = \pm 1$ distinguishes which of $\mathbf{3}$ or $\bar{\mathbf{3}}$ the solution belongs to. This matrix can readily be diagonalized, and the differential equation can be easily solved [25, 26]. The eigenvalues are

$$-1, 1, 1, 1, 1, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 2, \frac{7}{2}, \frac{7}{2} \quad (44)$$

if $s = +1$, and

$$-\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 2, 2, 2, 2, 4 \quad (45)$$

if $s = -1$. An intriguing feature of these two sets of eigenvalues is that they are mapped with each other by the reflection about $\frac{3}{2}$: $\lambda \mapsto 3 - \lambda$.

Suppose that λ is one of the eigenvalues of M above, and let η_λ be a corresponding (constant) eigenvector:

$$M\eta_\lambda = \lambda\eta_\lambda. \quad (46)$$

Then, writing

$$\tilde{\chi}_{6D}^+ = f_\lambda(x)\eta_\lambda \quad (47)$$

with some scale factor function $f_\lambda(x)$, $f_\lambda(x)$ is trivially solved to be

$$f_\lambda(x) = \text{constant} \times h(x)^{-\lambda} \quad (48)$$

so that

$$\tilde{\chi}_{6D}^+ = \text{constant} \times h(x)^{-\lambda} \eta_\lambda \quad (49)$$

or

$$\chi_{6D} = \text{constant} \times h(x)^{-\lambda+1} \begin{pmatrix} \eta_\lambda \\ 0 \end{pmatrix}. \quad (50)$$

In any zero-mode analysis, it is crucial to set appropriate boundary conditions for the solutions. In the present case, we are interested in the modes localized near $x = 0$, the locus of intersecting 5-branes with positive tension, and not near $x = \pm L$ where the negative tension ones reside. Thus we impose that the solution should vanish at $x = \pm L$. This boundary condition requires that the eigenvalues of M must be less than 1, which are -1 for $s = +1$ and two $-\frac{1}{2}$'s for $s = -1$ (FIG.3). Therefore, (calling the former “**3**” and the latter “**3**” of $SU(3)$) we find two chiral and one anti-chiral zero modes that satisfy the boundary condition, being in agreement with the prediction made by the $E_8/(E_6 \times SU(2) \times U(1))$ or $E_8/(E_6 \times U(1) \times U(1))$ supersymmetric sigma model.

D. Zero modes near the negative tension brane and (non)normalizability

Other eigenvalues λ , the ones except -1 in (44) and two $-\frac{1}{2}$'s in (45), are all larger than or equal to 1. The overall function in χ_{6D} blows up at $x = \pm L$ if $\lambda > 1$, or are constant if

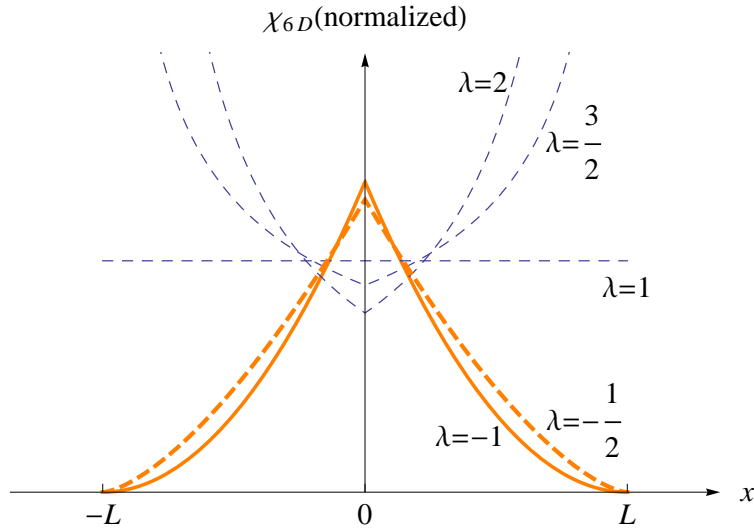


FIG. 3: The normalized zero mode profiles.

$\lambda = 1$.

On the background with the string frame metric (21), the volume form $d\mu$ of the transverse dimensions is given by

$$d\mu = dx^1 dx^2 \cdots dx^6 (h(x^1))^4. \quad (51)$$

If the delocalized dimensions x^2, \dots, x^6 are compactified on T^5 with a common radius R , then the measure becomes

$$d\mu = dx \cdot (2\pi R)^5 h(x)^4, \quad (52)$$

where $x^1 \equiv x$. Therefore, in order for χ_{6D} to be normalizable:

$$\int d\mu \chi_{6D}^\dagger \chi_{6D} < \infty, \quad (53)$$

the eigenvalue λ must satisfy

$$\lambda < \frac{7}{2}. \quad (54)$$

(If $\lambda = \frac{7}{2}$ the norm diverges logarithmically.) The modes that do not satisfy this condition is the two modes with $\lambda = \frac{7}{2}$ for $s = +1$, and the one with $\lambda = 4$ for $s = -1$. Thus there are three non-normalizable modes, two are anti-chiral and one is chiral. This leaves an equal number of chiral and anti-chiral zero modes that are *not* localized near $x = 0$.

In view of this, one might expect that they would be grouped into pairs and become massive. However, these remaining set of eigenvalues in (44) and (45) are not completely equal. For instance, (44) has four 1's, whereas (45) has only a single such mode. Since different eigenvalues correspond to different profiles, this fact makes it questionable whether these modes are all grouped into massive modes. If this $(2 - 1)$ -generation toy model is generalized to be more realistic for phenomenological applications, then the effect of these modes would also need to be seriously considered.

E. An orientifold substitute in heterotic string theory

In section III B, we considered a periodic harmonic function in the 5-brane solution so that one of the overall transverse dimensions be compactified. The periodic harmonic function necessarily led to the introduction of negative tension branes. In general, it is a well-known

fact that for warped compactification there must be some branes with negative tension [47], so what we have encountered above may be seen to be consistent with the no-go theorem.

In type II theories an orientifold can be identified as such a negative tension object. However, in heterotic string theory, the ordinary microscopic definition of orientifolds does not make sense because the heterotic string world sheet is asymmetric. How can we understand such a negative tension object in heterotic string theory?

This problem was considered in [48] (see also [49] for earlier discussions), where it was argued that a negative tension brane in heterotic string theory could be understood as a T-dual of the Atiyah-Hitchin [50] manifold. The argument is based on the fact that the metric of the Atiyah-Hitchin space asymptotically approaches that of the Taub-NUT space but with *negative* NUT charge at large distances. More precisely, the Atiyah-Hitchin metric can be written, by using elliptic theta functions, in the form

$$ds^2 = a^2 b^2 c^2 dt^2 + a^2 \sigma_1^2 + b^2 \sigma_2^2 + c^2 \sigma_3^2, \\ a^2 = \frac{w_2 w_3}{w_1}, \quad b^2 = \frac{w_3 w_1}{w_2}, \quad c^2 = \frac{w_1 w_2}{w_3}, \quad (55)$$

$$w_j = 2 \frac{d}{dt} \log \vartheta_{j+1}(0, 2\pi i m^2 t) \quad (j = 1, 2, 3) \quad (56)$$

with σ_1, σ_2 and σ_3 being the $SU(2)$ Maurer-Cartan 1-forms. If one ignores the rapidly vanishing terms containing $e^{-\frac{1}{2m^2 t}}$ in the theta functions, one can obtain the asymptotic behavior of the metric:

$$ds^2 \stackrel{t \rightarrow 0}{\sim} \frac{1 - 4m^2 t}{4m^2 t^4} dt^2 + \frac{4m^2}{1 - 4m^2 t} \sigma_3^2 + \frac{1 - 4m^2 t}{4m^2 t^2} (\sigma_1^2 + \sigma_2^2) \\ = \left(1 - \frac{2m}{r}\right) (dr^2 + r^2 (\sigma_1^2 + \sigma_2^2)) + 4m^2 \left(1 - \frac{2m}{r}\right)^{-1} \sigma_3^2, \quad (57)$$

where $r = \frac{1}{2mt}$ ($t > 0, m > 0$). This is the negative-charge Taub-NUT. While the Atiyah-Hitchin space is a smooth manifold, a singularity at $r = 2m$ has arisen in the asymptotic form (57) because the infinitely many exponential terms has been discarded. From the probe gauge theory point of view, these terms are regarded as the instanton corrections [48].

The Atiyah-Hitchin metric satisfies Einstein's equation, and so does the asymptotic form (57). One can embed (57) in ten dimensions as a part of a string frame metric. Then by taking T-duality along the Hopf fiber direction of the Taub-NUT, we obtain

$$ds_{\text{T-dual}}^2 = \eta_{ij} dx^i dx^j + V(r) \delta_{\mu\nu} dx^\mu dx^\nu,$$

$$\begin{aligned}
e^{2\Phi^{\text{T-dual}}} &= V(r), \\
H_{\mu\nu 9}^{\text{T-dual}} &= \epsilon_{\mu\nu 9}^{\rho} \partial_{\rho} V(r)
\end{aligned}
\tag{58}$$

where

$$V(r) = 1 - \frac{2m}{r} \tag{59}$$

with r being the *three*-dimensional radial coordinate here. Note that the ordinary smeared 5-brane has $V(r) = 1 + \frac{2m}{r}$. By further delocalizations except one direction the three-dimensional harmonic function $V(r)$ is replaced with

$$1 - \frac{2m}{r} \rightarrow 1 - 2m|x|. \tag{60}$$

Since the smeared intersecting solution (21) can be viewed as a superposition of two smeared solutions of the form (58), the negative tension objects located at $x = \pm L(+2nL)$ (FIG.2) could be thought of as a superposition of two T-duals of the Atiyah-Hitchin manifold [48].

While negative tension branes in heterotic string theory sounds bizarre, heterotic strings propagating on the celebrated Atiyah-Hitchin hyper-Kähler manifold will be no problem. Therefore, if a smooth T-dual (or mirror) of the Atiyah-Hitchin space exists, the negative tension branes will be asymptotic approximation of it at large distances. Also, since a T-dual of two intersecting 5-branes (with positive tensions) is a conifold [40, 51], it would be interesting to explore what corresponds to the Atiyah-Hitchin manifold in six dimensions.

An important aspect of this identification is that the transverse space is *forced* to be \mathbf{Z}_2 -orbifolded about the loci of the negative tension branes, similarly to the ordinary orientifolds in type II theories. This is because the Atiyah-Hitchin manifold has a so-called “bolt” [52]

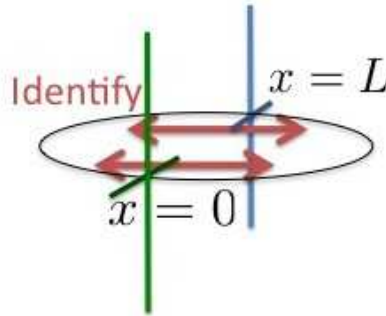


FIG. 4: The \mathbf{Z}_2 identification due to the “bolt”.

at the center [53]. Therefore, although at the beginning we started from the $x(=x^1)$ space compactified on a circle by taking a periodic array (24), the circle is necessarily subject to the \mathbf{Z}_2 identifications

$$x \sim 2(2n+1)L - x \quad (n \in \mathbf{Z}), \quad (61)$$

so that the circle is orbifolded into an interval, which is obligatory for our setup (FIG.4). This is in contrast to what is usually done in the gauge Higgs unification [54] or orbifold GUT theories [43, 44], where the \mathbf{Z}_2 projection is imposed *by hand*. Thus our intersecting 5-brane scenario may provide a natural origin of the \mathbf{Z}_2 orbifoldization in the extra dimensions, which has been simply postulated in those bottom-up approaches.

IV. INTERSECTING 5-BRANES WITH WILSON LINES

In the previous section, we have seen that the intersecting 5-branes with the $SU(3)$ gauge field obtained by the standard embedding support two chiral and one anti-chiral fermionic zero modes, which agrees with the prediction of the supersymmetric nonlinear sigma model on $E_8/(E_6 \times U(1) \times U(1))$ (or $E_8/(E_6 \times SU(2) \times U(1))$). In order to realize more realistic models with an unbroken $SO(10)$ or $SU(5)$ gauge group, we need to consider the gauge field configuration larger than $SU(3)$. This amounts to considering a non-standard embedding version of the symmetric 5-brane and their intersection, which is not known at present. Therefore, as a preliminary investigation into aspects of the non-standardly embedded branes, we consider in this section a constant Wilson line in the transverse space and see what happens to the profiles of the fermion zero modes. A discussion on more general gauge configurations will be given in the Summary and Discussion section.

Let us consider a constant $U(1)$ Wilson line proportional to h_\perp ($= \frac{1}{3}(-(E_1^1 + \cdots E_5^5) + 2E_6^6 + (E_7^7 + E_8^8 + E_9^9))$) (A15) in the x^2 component of A_M in addition to the $SU(3)$ gauge field (27), where we take the generators for the latter to be X_{ab} , Y_{ab} and h_a (A12). The Wilson line breaks the E_6 gauge symmetry to $SO(10) \times U(1)$. Of course, being constant, it does not affect the Bianchi identity (15). According to the decomposition of the E_6 adjoint

$$\mathbf{78} = \mathbf{45}_0 \oplus \mathbf{16}_1 \oplus \overline{\mathbf{16}}_{-1} \oplus \mathbf{1}_0 \quad (62)$$

as $SO(10) \times U(1)_\perp$ representations, the E_6 adjoint gaugino gives rise to a **16** and a $\overline{\mathbf{16}}$ $SO(10)$ gauginos. They have opposite $U(1)_\perp$ charges. We write

$$A_\mu^{U(1)} = iw|x|'\delta_\mu^2 \quad (63)$$

with a real constant w , where $|x|' = \frac{d}{dx}|x| = \pm 1$. The equation of motion for the **16** gaugino

$$\gamma^\mu \left(\partial_\mu + \frac{1}{4} \left(\omega - \frac{H}{3} \right)_{\mu\alpha\beta} \gamma^{\alpha\beta} + A_\mu^{U(1)} \right) \tilde{\chi}_{6D} = 0. \quad (64)$$

is reduced to

$$h^{-1} \begin{pmatrix} & -i\partial_{x^1} \\ i\partial_{x^1} & \end{pmatrix} + \frac{1}{4} \left(\omega + \frac{H}{3} \right)_{\tilde{\alpha}\beta\gamma} \gamma^{\tilde{\alpha}\beta\gamma} + iw|x|'h^{-1} \begin{pmatrix} & \tilde{\gamma}^2 \\ \tilde{\gamma}^2 & \end{pmatrix} = 0. \quad (65)$$

The one for $\overline{\mathbf{16}}$ can be obtained by flipping the sign of w (63). Assuming again that $\tilde{\chi}_{6D}$ has only the upper component $\tilde{\chi}_{6D}^+$ like (39), the problem amounts to solving

$$\left(\frac{d}{dx} + M^{U(1)} \right) \tilde{\chi}_{6D}^+ = 0, \quad (66)$$

where $M^{U(1)}$ is this time a 4×4 matrix

$$M^{U(1)} = \begin{pmatrix} \frac{3\alpha}{2} & -\frac{\alpha}{4} & c & -\frac{\alpha}{4} \\ -\frac{\alpha}{4} & \frac{3\alpha}{2} & -\frac{\alpha}{4} & c \\ c & -\frac{\alpha}{4} & \frac{3\alpha}{2} & -\frac{\alpha}{4} \\ -\frac{\alpha}{4} & c & -\frac{\alpha}{4} & \frac{3\alpha}{2} \end{pmatrix} \quad (67)$$

with $\alpha = \frac{h'}{h}$ and $c = w|x|'$. This matrix can be diagonalized by a *constant* matrix V :

$$V^{-1}M^{U(1)}V = \text{diag} \left\{ \alpha + c, 2\alpha + c, \frac{3\alpha}{2} - c, \frac{3\alpha}{2} - c \right\}, \quad (68)$$

$$V = \begin{pmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}. \quad (69)$$

Note that this simplification does not occur when the Wilson line is taken in any of the x^3, \dots, x^6 directions. This is one of the reasons why we have chosen x^2 as the direction of the Wilson line. (Also if x^1 is chosen, the (correspondingly modified) matrix $M^{U(1)}$ is again diagonalized by a constant matrix.) In the present case, (66) is thus easily solved: Let

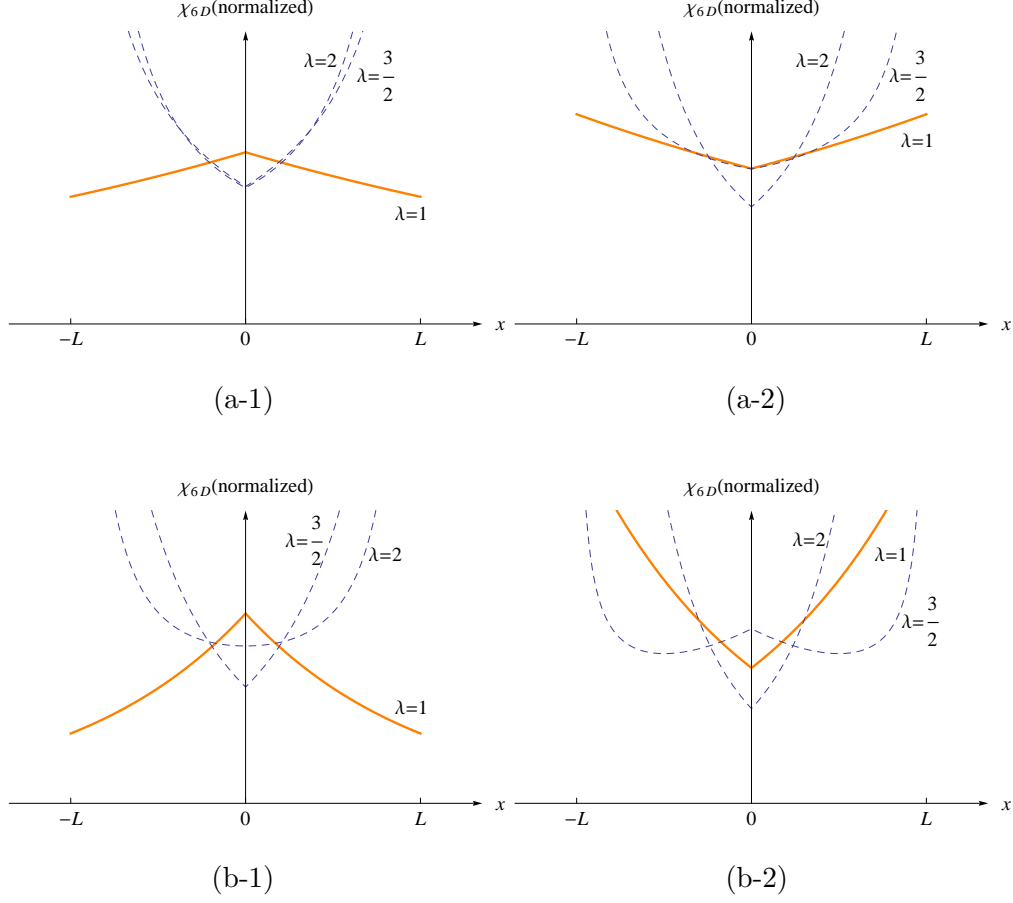


FIG. 5: The normalized $U(1)_\perp$ -charged zero mode profiles. The cases for $w = 0.3, -0.3, +1$ and -1 are shown in (a-1), (a-2), (b-1) and (b-2), respectively.

$\lambda = 1, 2, \frac{3}{2}$ be the coefficient of α of each eigenvalue, the unnormalized wave functions are given by

$$\chi_{6D}^{\lambda, \text{unnorm.}} = h(x)^{-\lambda+1} e^{-(-1)^{2\lambda} w |x|} \eta_\lambda, \quad (70)$$

where η_λ is the corresponding column vector of V . Taking the measure (52) into account, the profiles (scale factors) of the normalized zero mode wave functions are shown in FIG.5 for $w = \pm 0.3$ and $w = \pm 1$.

Let us now compare this result with the coset sigma models. In the present case, the relevant coset is $E_8/(SO(10) \times H \times U(1))$ with H being $SU(3)$ or $SU(2) \times U(1)$ or $U(1) \times U(1)$. It is not obvious how the transverse gauge configuration is related to the $U(1)$ Y -charge of the sigma model. In any case, however, one can have at most three chiral generations with an anti-chiral one whatever option of Y -charge is made, as we wrote in section II C. Therefore, since we already have two chiral and one anti-chiral $SU(3)$ -charged zero modes, another

($U(1)_\perp$ -charged) chiral zero mode should appear if the correspondence to the sigma model holds true.

In FIG.5 we can see that the $\lambda = 1$ curve changes its shape considerably depending on the sign of the Wilson line, while the curves for $\lambda = 2$ and $\lambda = \frac{3}{2}$ do not. In particular, only when $\lambda = 1$ and $w > 0$ the profile has the maximum at $x = 0$ and monotonically decreases as $x \rightarrow \pm L$ [66]. However, it is hard to regard this mode as the expected $U(1)_\perp$ -charged zero mode because the one with opposite chirality ($\lambda = 1$, $w < 0$) localizes near the other brane at $x = \pm L$. Thus we conclude that the inclusion of a constant Wilson line only has the effect of splitting the chiral and anti-chiral components apart and localizing them to regions near the different branes, creating no net chiral fermions in the four-dimensional sense. This means that a constant Wilson line is not enough to realize the $SO(10)$ theory in this setup but we need to do something else.

V. SUMMARY AND DISCUSSION

Motivated by the fact that some exceptional supersymmetric coset sigma models yield a set of matter fields close to what we really observe in nature, we have proposed a possible scenario of realizing such a sigma model in heterotic string theory by using NS5-branes. We have considered a system of two stacks of intersecting NS5-branes and identified as the one realizing the $E_8/(E_6 \times SU(2) \times U(1))$ (or $E_8/(E_6 \times U(1) \times U(1))$) sigma model. We have examined the validity of the identification by explicitly computing the chiral zero modes on the smeared background with all the transverse dimensions being compactified except one extra dimension. We have found two chiral and one anti-chiral zero modes which have their maximum at the location of the positive tension brane and vanish at the negative tension brane. This result is consistent with the sigma model prediction, confirming the validity of the identification.

We have also performed a similar computation with including a constant Wilson line in one of the transverse dimensions, and compared with the spectrum with the $E_8/(SO(10) \times H \times U(1))$ sigma model. In this case as well, we have indeed found a pair of chiral and anti-chiral modes which are asymmetrically localized on different branes, but it does not seem likely that they can be used as the realization of a new chiral generation. In order to achieve a more realistic coset like $E_8/(SO(10) \times SU(3) \times U(1))$ or $E_8/(SO(5) \times SU(3) \times U(1) \times U(1))$,

a non-standard embedding analogue of the NS5-brane will be needed.

In the presence of H fluxes, the Killing spinor equation (25) ensures that $\omega - H$ is in $SU(3)$, but says nothing about $\omega + H$. If $dH = 0$, it is the latter that is set equal to the gauge connection in the “standard” embedding, and not the former. Therefore, the gauge configuration determined by setting equal to $\omega + H$ *need not* be in $SU(3)$ in the presence of fluxes, but is generically in $SO(6)(= SU(4))$. This is in contrast to the case without H fluxes, where there is no distinction between these two $(\omega \pm H)$ connections. Nevertheless, for the intersecting 5-brane solution (21) not only $\omega - H$ but also $\omega + H$ belongs to $SU(3)$.

For smooth Calabi-Yau manifolds without H flux, it has been known for a long time that the Donaldson-Uhlenbeck-Yau theorem ensures the existence of an $SU(4)$ gauge field that preserves SUSY for every complex structure of a stable holomorphic vector bundle [37, 55], and it was suggested that such a connection could be obtained by first considering a direct sum of the $SU(3)$ bundle and a trivial line bundle, and then deforming the complex structure [55]. It would be natural to ask whether an analogous theorem holds in the presence of H fluxes. If only the existence of such a brane configuration could be ensured, one could in principle use the nonlinear sigma model to describe low-energy physics.

We have argued that the negative tension brane needed in the global model could be understood as a superposition of T-duals of the Atiyah-Hitchin manifold. The setup has been reduced to a system similar to that postulated in the gauge-Higgs unification scenario, where the Z_2 orbifold structure is naturally inherited from the bolt singularity in the Atiyah-Hitchin manifold. It would be interesting to study the Hosotani mechanism in this setup. Also, a microscopic understanding of the system of intersecting NS5-branes is desirable. Since it is T-dual to the conifold, the noncompact Gepner model approach [56] might shed light on this issue.

The original idea of identifying the observed fermions as coming from a “preon” theory suffered from the following problems: First, now that the Higgs boson seems to have been finally found, there is no evidence for the composite model. Secondly, the origins of the gauge and gravitational interactions are unclear. And finally, Kugo-Yanagida’s E_7 model and its E_8 generalization with an unbroken $SU(5)$ are anomalous. One of the virtues of the “quasi-Nambu-Goldstone fermion hypothesis in string theory” proposed in this paper is that it can solve, or at least give a possible scenario to solve, all of these problems. Indeed, since we use a symmetry breaking due to branes, the preon theory is not needed any more. Gauge

interactions and gravity are, of course, built in heterotic string theory. Also, the anomaly may be basically canceled by an anomaly inflow from the bulk.

This last point requires further discussion. The anomaly polynomial for a chiral fermion in four-dimensional Yang-Mills theory is $\frac{1}{48\pi^3}\text{Tr}F^3$. At first sight, the $\text{Tr}F^4$ term in X_8 in the Green-Schwarz counter term may appear to yield the requisite contribution if $\text{Tr}F^4$ contains a term of the form $\text{Tr}F^3 \wedge F_{U(1)}$ in a decomposition in terms of some subgroup of E_8 and the $F_{U(1)}$ develops an expectation value in the transverse space. However, this is not the case because $\text{Tr}F^4 \propto (\text{Tr}F^2)^2$ for E_8 . This means that one requires yet another gauge-variant term besides the ordinary Green-Schwarz counter term in order to cancel the $SU(5)$ anomaly by the anomaly inflow. Although such a correction of the supergravity background itself is not surprising in the $(2,0)$ worldsheet SUSY models [57, 58], one would need an explicit form of the field configuration (and not just the existence of it) in order to confirm the cancellation by a direct computation.

There are many questions to be asked: How is SUSY broken and mediated? How is the GUT group broken? What is the origin of the Yukawa hierarchy? How are the dilaton and other moduli stabilized? Also, although likely to exist, we haven't so far found an explicit 5-brane field configuration that realizes three families of fermions. We emphasize, however, that the “quasi-Nambu-Goldstone fermion hypothesis in string theory” proposed in this paper is unprecedented in that it might at least provide a scenario *explaining why there are three* (or rather, not larger than four or five), without resorting to the help of the anthropic principle. In view of the fact that there seems no other viable explanation for it, we believe our new framework will deserve further investigations.

Appendix A: Generators of E_8 and its subalgebras

In this appendix we give some detail about E_8 , in particular in terms of Freudenthal's realization, which uses the decomposition of E_8 into representations of the maximal subgroup $SL(9)$. Although it is more familiar to realize E_8 as a direct sum of the $SO(16)$ adjoint and spinor representations, Freudenthal's realization has an advantage in that the generators have only the $SL(9)$ vector indices. It was used by Irié and Yasui [20] to describe the Kähler coset $E_8/(SO(10) \times SU(3) \times U(1))$ [67].

1. $E_{8(+8)}$

$$\begin{aligned}
E^I_J & \quad (I, J = 1, \dots, 9; \quad I \neq J) & (\text{total } 72) \\
E^{IJK} & \quad (I, J, K = 1, \dots, 9) & (\text{total } 84) \\
E^*_{IJK} & \quad (I, J, K = 1, \dots, 9) & (\text{total } 84) \\
h_I & \quad (I = 1, \dots, 8) \quad (= E^I_I - E^J_J) & (\text{total } 8)
\end{aligned} \tag{A1}$$

$$h_{IJK} \equiv E^I_I + E^J_J + E^K_K - \frac{1}{3} \sum_{L=1}^9 E^L_L \tag{A2}$$

$$\begin{aligned}
[E^I_J, E^K_L] &= \delta^K_J E^I_L - \delta^I_L E^K_J, \\
[E^I_J, E^{KLM}] &= 3\delta^{[M}_I E^{KL]J}, \\
[E^I_J, E^*_{KLM}] &= -3\delta^I_{[M} E^*_{KL]J}, \\
[E^{IJK}, E^{LMN}] &= -\frac{1}{3!} \sum_{P,Q,R=1}^9 \epsilon^{IJKLMNPQR} E^*_{PQR}, \\
[E^*_{IJK}, E^*_{LMN}] &= +\frac{1}{3!} \sum_{P,Q,R=1}^9 \epsilon_{IJKLMNPQR} E^{PQR}, \\
[E^{IJK}, E^*_{LMN}] &= 6\delta^J_{[M} \delta^K_N E^I_{L]} \quad (\text{if } I \neq L, M, N), \\
[E^{IJK}, E^*_{IJK}] &= h_{IJK}.
\end{aligned} \tag{A3}$$

$$\begin{aligned}
\text{Tr}_{\mathbf{248}} E^I_J E^K_L &= 60 \delta^I_L \delta^K_J, \\
\text{Tr}_{\mathbf{248}} E^{IJK} E^*_{LMN} &= 60 \cdot 6 \delta^I_{[L} \delta^J_M \delta^K_{N]}.
\end{aligned} \tag{A4}$$

2. The compact E_8 ($= E_{8(-248)}$)

$$\begin{aligned}
X_{IJ} &\equiv E^I_J + E^J_I & (1 \leq I < J \leq 9) \\
Y_{IJ} &\equiv -i(E^I_J - E^J_I) & (1 \leq I < J \leq 9) \\
h_I &\equiv E^I_I - E^{I+1}_{I+1} & (I = 1, \dots, 8) \\
X_{IJK} &\equiv E^{IJK} + E^*_{IJK} & (1 \leq I < J < K \leq 9) \\
Y_{IJK} &\equiv -i(E^{IJK} - E^*_{IJK}) & (1 \leq I < J < K \leq 9)
\end{aligned} \tag{A5}$$

$$\begin{aligned}
[X_{IJ}, X_{KL}] &= +i(\delta_{JK}Y_{IL} + \delta_{IL}Y_{JK} + \delta_{IK}Y_{JL} + \delta_{JL}Y_{IK}), \\
[Y_{IJ}, Y_{KL}] &= -i(\delta_{JK}Y_{IL} + \delta_{IL}Y_{JK} - \delta_{IK}Y_{JL} - \delta_{JL}Y_{IK}), \\
[X_{IJ}, Y_{KL}] &= -i(\delta_{JK}X_{IL} - \delta_{IL}X_{JK} + \delta_{IK}X_{JL} - \delta_{JL}X_{IK}), \\
[Y_{IJ}, X_{KL}] &= -i(\delta_{JK}X_{IL} - \delta_{IL}X_{JK} - \delta_{IK}X_{JL} + \delta_{JL}X_{IK}), \\
[X_{IJ}, X_{KLM}] &= +3i(\delta_{J[M}Y_{KL]I} + \delta_{I[M}Y_{KL]J}), \\
[Y_{IJ}, X_{KLM}] &= -3i(\delta_{J[M}X_{KL]I} - \delta_{I[M}X_{KL]J}), \\
[X_{IJ}, Y_{KLM}] &= -3i(\delta_{J[M}X_{KL]I} + \delta_{I[M}X_{KL]J}), \\
[Y_{IJ}, Y_{KLM}] &= -3i(\delta_{J[M}Y_{KL]I} - \delta_{I[M}Y_{KL]J}), \\
[X_{IJK}, X_{LMN}] &= +\frac{i}{6}\sum_{P,Q,R=1}^9 \epsilon_{IJKLMNPQR}Y_{PQR} + 18i\delta_{JM}\delta_{KN}Y_{IL} \quad ([IJK],[LMN]), \\
[Y_{IJK}, Y_{LMN}] &= -\frac{i}{6}\sum_{P,Q,R=1}^9 \epsilon_{IJKLMNPQR}Y_{PQR} + 18i\delta_{JM}\delta_{KN}Y_{IL} \quad ([IJK],[LMN]), \\
[X_{IJK}, Y_{LMN}] &= +\frac{i}{6}\sum_{P,Q,R=1}^9 \epsilon_{IJKLMNPQR}X_{PQR} + 18i\delta_{JM}\delta_{KN}X_{IL} \\
&\quad - 2i\delta_{JM}\delta_{KN}\delta_{IL}\sum_{P=1}^9 X_{PP} \quad ([IJK],[LMN]),
\end{aligned} \tag{A6}$$

In the last line, the trace part of X is projected out, and $X_{II} - X_{I+1 \ I+1}$ are identified as $2h_I$. The symbol $([IJK],[LMN])$ means that the rhs must be anti-symmetrized in the indicated fashion.

3. $E_8 \supset E_7 \times SU(2)$

$$\mathbf{248} = (\mathbf{133}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{56}, \mathbf{2})$$

In the following decompositions, the denominator subalgebras are compact ones, whereas the coset generators are expressed in the $E_{8(+8)}$ form.

- $(133, 1)$ ($= E_7$)

$$\begin{aligned}
X_{\hat{i}\hat{j}} & \quad (1 \leq \hat{i} < \hat{j} \leq 7) && \text{(total 21)} \\
Y_{\hat{i}\hat{j}} & \quad (1 \leq \hat{i} < \hat{j} \leq 7) && \text{(total 21)} \\
h_{\hat{i}89} & \quad (\hat{i} = 1, \dots, 7) && \text{(total 7)} \\
X_{\hat{i}89} & \quad (\hat{i} = 1, \dots, 7) && \text{(total 7)} \\
Y_{\hat{i}89} & \quad (\hat{i} = 1, \dots, 7) && \text{(total 7)}
\end{aligned} \tag{A7}$$

$E_{\hat{j}}^{i_{\hat{j}}(\hat{i} \neq \hat{j})} E^{\hat{i}89}_{i89} E^*_{i89} h_{i89} E^{\hat{i}\hat{j}\hat{k}} E^*_{i\hat{j}\hat{k}}$									
$(\mathbf{133}, \mathbf{1})$									
$E^i_j E^{i89}_{i89} E^*_{i89} h_{i89}$ $(\mathbf{24}, \mathbf{1}, \mathbf{1})$ 0	$E^a_b h_{\bar{a}}$ $(\mathbf{1}, \mathbf{3}, \mathbf{1})$ 0	h_{\sharp} $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ 0	$E^i_a E^{a89}_{a89}$ $(\mathbf{5}, \mathbf{\bar{3}}, \mathbf{1})$ -4	$E^a_i E^{a89}_i$ $(\mathbf{\bar{5}}, \mathbf{3}, \mathbf{1})$ 4	$E^*_{ijk} E^{567}_{567}$ $(\mathbf{5}, \mathbf{1}, \mathbf{1})$ 6	$E^{ijk} E^*_{567}$ $(\mathbf{\bar{5}}, \mathbf{1}, \mathbf{1})$ -6	$E^*_{ija} E^{iab}$ $(\mathbf{10}, \mathbf{\bar{3}}, \mathbf{1})$ 2	$E^{ija} E^*_{iab}$ $(\mathbf{\bar{10}}, \mathbf{3}, \mathbf{1})$ -2	

$$h_{\sharp} = -2(E^1_1 + \dots + E^4_4) + 2(E^5_5 + E^6_6 + E^7_7) + E^8_8 + E^9_9. \quad (\text{A10})$$

In *this* table, the values that the various indices run over are: $\hat{i}, \hat{j}, \hat{k} = 1, \dots, 7$; $i, j, k = 1, \dots, 4$; $a, b = 5, 6, 7$; $\bar{a} = 5, 6$.

5. $E_8 \supset E_6 \times SU(3)$

$$\mathbf{248} = (\mathbf{78}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \oplus (\mathbf{27}, \mathbf{3}) \oplus (\mathbf{\bar{27}}, \mathbf{\bar{3}})$$

• $(\mathbf{78}, \mathbf{1}) (= E_6)$

$$X_{ij} \quad (1 \leq i < j \leq 6) \quad (\text{total } 15)$$

$$Y_{ij} \quad (1 \leq i < j \leq 6) \quad (\text{total } 15)$$

$$h_{\bar{i}} \quad (\bar{i} = 1, \dots, 5) \quad (\text{total } 5)$$

$$X_{789} \quad (\text{total } 1)$$

$$Y_{789} \quad (\text{total } 1)$$

$$h_{789} \quad (\text{total } 1)$$

(A11)

$$X_{ijk} \quad (1 \leq i < j < k \leq 6) \quad (\text{total } 20)$$

$$Y_{ijk} \quad (1 \leq i < j < k \leq 6) \quad (\text{total } 20)$$

• $(\mathbf{1}, \mathbf{8}) (= SU(3))$

$$X_{ab} \quad (7 \leq a < b \leq 9) \quad (\text{total } 3)$$

$$Y_{ab} \quad (7 \leq a < b \leq 9) \quad (\text{total } 3)$$

$$h_{\bar{a}} \quad (\bar{a} = 7, 8) \quad (\text{total } 2)$$

(A12)

• **(27, 3)**

$$\begin{aligned}
E_i^a & \quad (i = 1, \dots, 6; \ a = 7, 8, 9) & \text{(total } 6 \times 3) \\
E_{iab}^* & \quad (i = 1, \dots, 6; \ 7 \leq a < b \leq 9) & \text{(total } 6 \times 3) \\
E^{ija} & \quad (1 \leq i < j \leq 6; \ a = 7, 8, 9) & \text{(total } 15 \times 3)
\end{aligned} \tag{A13}$$

• **($\overline{27}, \overline{3}$)**

$$\begin{aligned}
E_a^i & \quad (i = 1, \dots, 6; \ a = 7, 8, 9) & \text{(total } 6 \times 3) \\
E^{iab} & \quad (i = 1, \dots, 6; \ 7 \leq a < b \leq 9) & \text{(total } 6 \times 3) \\
E_{ija}^* & \quad (1 \leq i < j \leq 6; \ a = 7, 8, 9) & \text{(total } 15 \times 3)
\end{aligned} \tag{A14}$$

$$\begin{aligned}
\mathbf{6.} \quad E_8(\supset E_6 \times SU(3)) & \supset (SO(10) \times U(1)) \times SU(3) \\
& \supset (SU(5) \times U(1) \times U(1)) \times SU(3)
\end{aligned}$$

Generators $E_6 \times SU(3)$ representations	$E_j^i (i \neq j) \ h_{\bar{i}} \ E^{ijk} \ E_{ijk}^* \ E^{789} \ E_{789}^*$ (78, 1)											
Generators $SU(5) \times SU(3)$ representations	$E_{\bar{j}}^{\bar{i}} \ h_{\bar{i}}$	$E^{\bar{i}\bar{j}6}$	$E_{\bar{i}\bar{j}6}^*$	h_{456}	$E_{\bar{i}\bar{j}\bar{k}}^*$	$E_{\bar{i}}^6$	E^{789}	$E^{\bar{i}\bar{j}\bar{k}}$	$E_{\bar{6}}^{\bar{i}}$	E_{789}^*	h_{\perp}	
$SO(10) \times SU(3)$ representations	(24,1)	(10,1)	($\overline{10}$,1)	(1,1)	(10,1)	($\bar{5}$,1)	(1,1)	($\overline{10}$,1)	(5,1)	(1,1)	(1,1)	
		(45,1)				(16,1)			($\overline{16}$,1)		(1,1)	
$3h_{\perp}$ charge		0				3			-3		0	
$3h'_{\perp}$ charge	0	3	-3	0	0	3	-3	0	-3	3	0	

(Cont'd)

$E_i^a \ E_{iab}^* \ E^{ija}$ (27, 3)						$E_a^i \ E^{iab} \ E_{ija}^*$ ($\overline{27}, \overline{3}$)						$E_b^a \ h_{\bar{a}}$ (1, 8)
$E^{\bar{i}\bar{j}a}$	E_{iab}^*	E_6^a	$E^{\bar{i}6a}$	$E_{\bar{i}}^a$	E_{6ab}^*	$E_{\bar{i}\bar{j}a}^*$	$E^{\bar{i}ab}$	E_a^6	$E_{\bar{i}6a}^*$	$E_{\bar{a}}^{\bar{i}}$	E^{6ab}	$E_b^a \ h_{\bar{a}}$
(10,3)	($\bar{5}$,3)	(1,3)	(5,3)	($\bar{5}$,3)	(1,3)	($\overline{10}$,3)	(5,3)	(1,3)	($\bar{5}$,3)	(5,3)	(1,3)	(1,8)
(16,3)			(10,3)	(1,3)		($\overline{16}$,3)			(10,3)	(1,3)	(1,3)	(1,8)
-1			2	-4		1			-2	4		0
-1	2	-4	2	-1	-1	1	-2	4	-2	1	1	0

$$h_{\perp} = \frac{1}{3} \left(-(E_1^1 + \dots + E_5^5) + 2E_6^6 + (E_7^7 + E_8^8 + E_9^9) \right). \tag{A15}$$

$$h'_\perp = E_6 - \frac{1}{3}(E_7 + E_8 + E_9). \quad (\text{A16})$$

Appendix B: Conventions for the gamma matrices

$$\Gamma^a = \gamma_{4D}^a \otimes \mathbf{1}, \quad (\text{B1})$$

$$\Gamma^\alpha = \gamma_{4D}^\# \otimes \gamma^\alpha, \quad (\text{B2})$$

where $a = 0, 7, 8, 9$ and $\alpha = 1, 2, 3, 4, 5, 6$.

$$\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}, \quad (\text{B3})$$

$$\eta^{AB} = \text{diag}(- + \cdots +).$$

$$\begin{aligned} \gamma_{4D}^0 &= i\sigma_2 \otimes \mathbf{1}, \\ \gamma_{4D}^7 &= \sigma_1 \otimes \sigma_1, \\ \gamma_{4D}^8 &= \sigma_1 \otimes \sigma_2, \\ \gamma_{4D}^9 &= \sigma_1 \otimes \sigma_3, \\ \gamma_{4D}^\# &= \sigma_3 \otimes \mathbf{1} = -i\gamma_{4D}^0\gamma_{4D}^7\gamma_{4D}^8\gamma_{4D}^9. \end{aligned} \quad (\text{B4})$$

$$\begin{aligned} \gamma^1 &= \sigma_2 \otimes \mathbf{1} \otimes \mathbf{1}, \\ \gamma^2 &= \sigma_1 \otimes \sigma_1 \otimes \mathbf{1}, \\ \gamma^3 &= \sigma_1 \otimes \sigma_2 \otimes \mathbf{1}, \\ \gamma^4 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_1, \\ \gamma^5 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_2, \\ \gamma^6 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_3, \\ \gamma^\# &= \sigma_3 \otimes \mathbf{1} \otimes \mathbf{1} = -i\gamma^1\gamma^2\cdots\gamma^6. \end{aligned} \quad (\text{B5})$$

$$\Gamma_{11} = \gamma_{4D}^\# \otimes \gamma_\# . \quad (\text{B6})$$

$$B = \Gamma^8\Gamma^1\Gamma^3\Gamma^5. \quad (\text{B7})$$

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- [62] However, most of the early family unification models suffered from the problem of mirror families. See [59] for more discussion.
- [63] For its anomalies see section V.
- [64] However, there is no cosmological constant in our setup, either in the bulk or on the branes. Also, warping is much smaller.
- [65] In fact, it is not obvious whether the existence of zero modes of the gaugino equation of motion, to be discussed in the next section, directly implies the existence of modes localized on the brane; according to the anomaly argument in section III A, the latter is supposed to live on the δ -function-like B-field source, while the former has a profile extended into the transverse dimensions. However, since the modes on the brane arise as a boundary contribution due to (20), it is natural to expect that the chirality of the bulk gaugino solution and that of the localized mode coincide, which we assume in this paper. Note, however, that this is the conventional interpretation of the zero mode in the literature (e.g. [29]).
- [66] The $SU(3)$ charged modes found in section III C are also charged with respect to $U(1)_\perp$, but their chiralities will not change as far as g_0 is small and hence A_μ 's (27) are large.
- [67] Incidentally, it is particularly useful to describe E_8 U-duality in three dimensions as the $SL(9)$

third-rank tensors correspond to the M theory 3-form [60, 61].